Section II: Free-Response

The following are examples of the kinds of free-response questions found on the exam.

PART A (AB OR BC)

A graphing calculator is required on this part of the exam.

t (hours)	0	2	4	6	8	10	12
$R(t)$ (vehicles per hour)	2935	3653	3442	3010	3604	1986	2201

1. On a certain weekday, the rate at which vehicles cross a bridge is modeled by the differentiable function R for $0 \le t \le 12$, where $R(t)$ is measured in vehicles per hour and t is the number of hours since 7:00 A.M. ($t = 0$). Values of $R(t)$ for selected values of t are given in the table above.

   (a) Use the data in the table to approximate $R'(5)$. Show the computations that lead to your answer. Using correct units, explain the meaning of $R'(5)$ in the context of the problem.

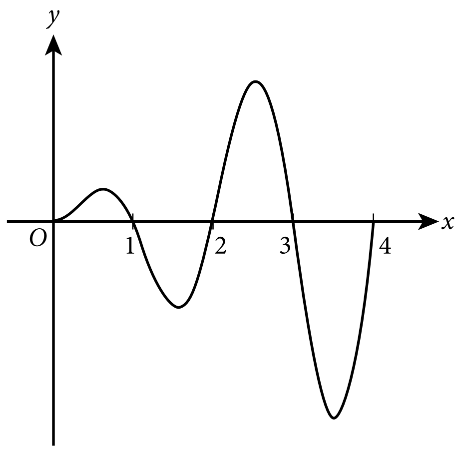
   (b) Use a midpoint sum with three subintervals of equal length indicated by the data in the table to approximate the value of $\int_0^{12} R(t)\,dt$. Indicate units of measure.

   (c) On a certain weekend day, the rate at which vehicles cross the bridge is modeled by the function H defined by $H(t) = -t^3 - 3t^2 + 288t + 1300$ for $0 \le t \le 17$, where $H(t)$ is measured in vehicles per hour and t is the number of hours since 7:00 A.M. ($t = 0$). According to this model, what is the average number of vehicles crossing the bridge per hour on the weekend day for $0 \le t \le 12$?

   (d) For $12 < t < 17$, $L(t)$, the local linear approximation to the function H given in part (c) at $t = 12$, is a better model for the rate at which vehicles cross the bridge on the weekend day. Use $L(t)$ to find the time t, for $12 < t < 17$, at which the rate of vehicles crossing the bridge is 2000 vehicles per hour. Show the work that leads to your answer.

PART B (AB OR BC)

Graphing calculators are not permitted on this part of the exam.



Graph of f'

2. The figure above shows the graph of f', the derivative of a twice-differentiable function f, on the closed interval $[0, 4]$. The areas of the regions bounded by the graph of f' and the x-axis on the intervals $[0, 1]$, $[1, 2]$, $[2, 3]$, and $[3, 4]$ are 2, 6, 10, and 14, respectively. The graph of f' has horizontal tangents at $x = 0.6$, $x = 1.6$, $x = 2.5$, and $x = 3.5$. It is known that $f(2) = 5$.

   (a) On what open intervals contained in $(0, 4)$ is the graph of f both decreasing and concave down? Give a reason for your answer.

   (b) Find the absolute minimum value of f on the interval $[0, 4]$. Justify your answer.

   (c) Evaluate $\int_0^4 f(x)f'(x)dx$.

   (d) The function g is defined by $g(x) = x^3 f(x)$. Find $g'(2)$. Show the work that leads to your answer.

PART A (BC ONLY)

A graphing calculator is required on this part of the exam.

3. For $0 \le t \le 5$, a particle is moving along a curve so that its position at time t is $(x(t), y(t))$. At time $t = 1$, the particle is at position $(2, -7)$. It is known that $\dfrac{dx}{dt} = \sin\left(\dfrac{t}{t+3}\right)$ and $\dfrac{dy}{dt} = e^{\cos t}$.

   (a) Write an equation for the line tangent to the curve at the point $(2, -7)$.

   (b) Find the y-coordinate of the position of the particle at time $t = 4$.

   (c) Find the total distance traveled by the particle from time $t = 1$ to time $t = 4$.

   (d) Find the time at which the speed of the particle is 2.5. Find the acceleration vector of the particle at this time.

PART B (BC ONLY)

Graphing calculators are not permitted on this part of the exam.

4. The Maclaurin series for the function f is given by

   $f(x) = \sum_{k=1}^{\infty} \frac{(-1)^{k+1} x^k}{k^2} = x - \frac{x^2}{4} + \frac{x^3}{9} - \cdots$ on its interval of convergence.

   (a) Use the ratio test to determine the interval of convergence of the Maclaurin series for f. Show the work that leads to your answer.

   (b) The Maclaurin series for f evaluated at $x = \frac{1}{4}$ is an alternating series whose terms decrease in absolute value to 0. The approximation for $f\left(\frac{1}{4}\right)$ using the first two nonzero terms of this series is $\frac{15}{64}$. Show that this approximation differs from $f\left(\frac{1}{4}\right)$ by less than $\frac{1}{500}$.

   (c) Let h be the function defined by $h(x) = \int_0^x f(t)\,dt$. Write the first three nonzero terms and the general term of the Maclaurin series for h.