PART B (BC ONLY)

Graphing calculators are not permitted on this part of the exam.

4. The Maclaurin series for the function f is given by

$$f(x) = \sum_{k=1}^{\infty} \frac{(-1)^{k+1} x^k}{k^2} = x - \frac{x^2}{4} + \frac{x^3}{9} - \cdots \text{ on its interval of convergence.}$$

(a) Use the ratio test to determine the interval of convergence of the Maclaurin series for f. Show the work that leads to your answer.

(b) The Maclaurin series for f evaluated at $x = \frac{1}{4}$ is an alternating series whose terms decrease in absolute value to 0. The approximation for $f\left(\frac{1}{4}\right)$ using the first two nonzero terms of this series is $\frac{15}{64}$. Show that this approximation differs from $f\left(\frac{1}{4}\right)$ by less than $\frac{1}{500}$.

(c) Let h be the function defined by $h(x) = \int_0^x f(t)\,dt$. Write the first three nonzero terms and the general term of the Maclaurin series for h.

© 2019 College Board