Sample AP Calculus AB and BC Exam Questions

The sample exam questions that follow illustrate the relationship between the course framework and the AP Calculus AB and BC Exams and serve as examples of the types of questions that appear on the exams. After the sample questions is a table that shows which skill, learning objective(s), and unit each question relates to. The table also provides the answers to the multiple-choice questions.

Section I: Multiple-Choice

PART A (AB OR BC)

Graphing calculators are not permitted on this part of the exam.

1. $\lim_{x \to 0} \frac{1 - \cos^2(2x)}{(2x)^2} =$

   (A) 0

   (B) $\frac{1}{4}$

   (C) $\frac{1}{2}$

   (D) 1

$$f(x) = \begin{cases} \frac{2}{x} & \text{for } x < -1 \\ x^2 - 3 & \text{for } -1 \le x \le 2 \\ 4x - 3 & \text{for } x > 2 \end{cases}$$

2. Let f be the function defined above. At what values of x, if any, is f not differentiable?

   (A) $x = -1$ only

   (B) $x = 2$ only

   (C) $x = -1$ and $x = -2$

   (D) f is differentiable for all values of x.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	2	−4	−5	3
2	−3	1	8	4

3. The table above gives values of the differentiable functions f and g and their derivatives at selected values of x. If h is the function defined by $h(x) = f(x)g(x) + 2g(x)$, then $h'(1) =$

(A) 32

(B) 30

(C) −6

(D) −16

4. If $x^3 - 2xy + 3y^2 = 7$, then $\dfrac{dy}{dx} =$

(A) $\dfrac{3x^2 + 4y}{2x}$

(B) $\dfrac{3x^2 - 2y}{2x - 6y}$

(C) $\dfrac{3x^2}{2x - 6y}$

(D) $\dfrac{3x^2}{2 - 6y}$

5. The radius of a right circular cylinder is increasing at a rate of 2 units per second. The height of the cylinder is decreasing at a rate of 5 units per second. Which of the following expressions gives the rate at which the volume of the cylinder is changing with respect to time in terms of the radius r and height h of the cylinder?
(The volume V of a cylinder with radius r and height h is $V = \pi r^2 h$.)

(A) $-20\pi r$

(B) $-2\pi rh$

(C) $4\pi rh - 5\pi r^2$

(D) $4\pi rh + 5\pi r^2$

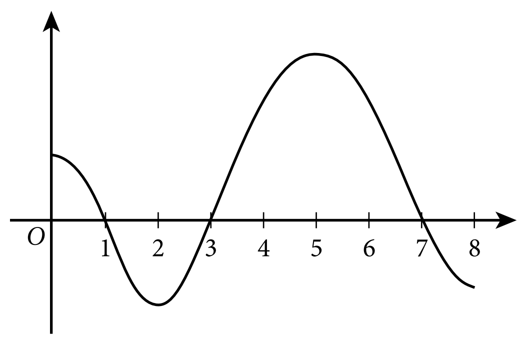
6. Which of the following is equivalent to the definite integral $\int_2^6 \sqrt{x}\,dx$?

(A) $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{4}{n}\sqrt{\frac{4k}{n}}$

(B) $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{6}{n}\sqrt{\frac{6k}{n}}$

(C) $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{4}{n}\sqrt{2+\frac{4k}{n}}$

(D) $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{6}{n}\sqrt{2+\frac{6k}{n}}$



Graph of g

7. The figure above shows the graph of the continuous function g on the interval $[0, 8]$. Let h be the function defined by $h(x) = \int_3^x g(t)\,dt$. On what intervals is h increasing?

(A) $[2, 5]$only

(B) $[1, 7]$

(C) $[0, 1]$ and $[3, 7]$

(D) $[1, 3]$ and $[7, 8]$

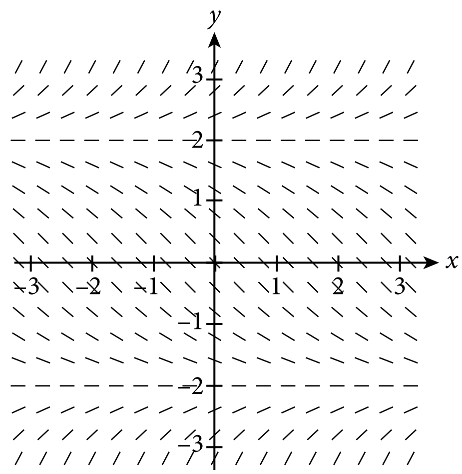
8. $\int \frac{x}{\sqrt{1-9x^2}}\,dx =$

(A) $-\frac{1}{9}\sqrt{1-9x^2} + C$

(B) $-\frac{1}{18}\ln\sqrt{1-9x^2} + C$

(C) $\frac{1}{3}\arcsin(3x) + C$

(D) $\frac{x}{3}\arcsin(3x) + C$



9. Shown above is a slope field for which of the following differential equations?

(A) $\dfrac{dy}{dx} = \dfrac{y-2}{2}$

(B) $\dfrac{dy}{dx} = \dfrac{y^2-4}{4}$

(C) $\dfrac{dy}{dx} = \dfrac{x-2}{2}$

(D) $\dfrac{dy}{dx} = \dfrac{x^2-4}{4}$

10. Let R be the region bounded by the graph of $x = e^y$, the vertical line $x = 10$, and the horizontal lines $y = 1$ and $y = 2$. Which of the following gives the area of R?

(A) $\int_1^2 e^y\,dy$

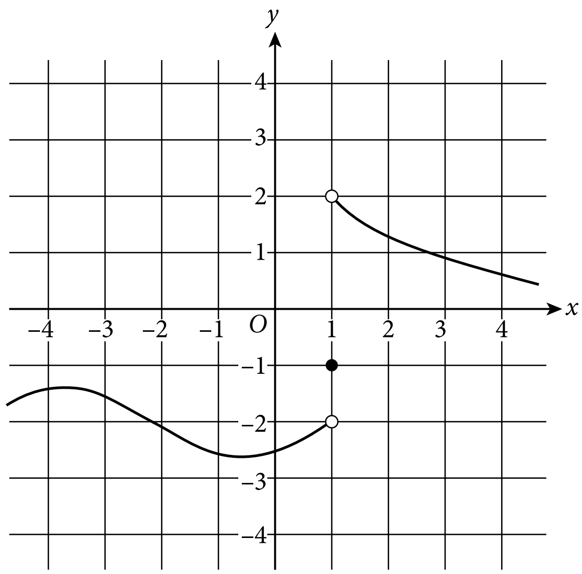
(B) $\int_e^{e^2} \ln x\,dx$

(C) $\int_1^2 (10 - e^y)\,dy$

(D) $\int_e^{10} (\ln x - 1)\,dx$

PART B (AB OR BC)

A graphing calculator is required on this part of the exam.



Graph of f

11. The graph of the function f is shown in the figure above. The value of $\lim_{x \to 1^+} f(x)$ is

(A) –2

(B) −1

(C) 2

(D) nonexistent

12. The velocity of a particle moving along a straight line is given by $v(t) = 1.3t\ln(0.2t + 0.4)$ for time $t \geq 0$. What is the acceleration of the particle at time $t = 1.2$?

(A) –0.580

(B) –0.548

(C) –0.093

(D) 0.660

x	−1	0	2	4	5
$f'(x)$	11	9	8	5	2

13. Let f be a twice-differentiable function. Values of f', the derivative of f, at selected values of x are given in the table above. Which of the following statements must be true?

(A) f is increasing for $-1 \le x \le 5$.

(B) The graph of f is concave down for $-1 < x < 5$.

(C) There exists c, where $-1 < c < 5$, such that $f'(c) = -\frac{3}{2}$.

(D) There exists c, where $-1 < c < 5$, such that $f''(c) = -\frac{3}{2}$.

14. Let f be the function with derivative defined by $f'(x) = 2 + (2x - 8)\sin(x + 3)$. How many points of inflection does the graph of f have on the interval $0 < x < 9$?

(A) One

(B) Two

(C) Three

(D) Four

15. Honey is poured through a funnel at a rate of $r(t) = 4e^{-0.35t}$ ounces per minute, where t is measured in minutes. How many ounces of honey are poured through the funnel from time $t = 0$ to time $t = 3$?

(A) 0.910

(B) 1.400

(C) 2.600

(D) 7.429

PART A (BC ONLY)

Graphing calculators are not permitted on this part of the exam.

x	2	5
$f(x)$	4	7
$f'(x)$	2	3

16. The table above gives values of the differentiable function f and its derivative f' at selected values of x.

If $\int_2^5 f(x)dx = 14$, what is the value of $\int_2^5 x \cdot f'(x)dx$?

(A) 13

(B) 27

(C) $\frac{63}{2}$

(D) 41

17. The number of fish in a lake is modeled by the function F that satisfies the logistic differential equation $\frac{dF}{dt} = 0.04F\left(1 - \frac{F}{5000}\right)$, where t is the time in months and $F(0) = 2000$. What is $\lim_{t \to \infty} F(t)$?

(A) 10,000

(B) 5000

(C) 2500

(D) 2000

18. A curve is defined by the parametric equations $x(t) = t^2 + 3$ and $y(t) = \sin(t^2)$. Which of the following is an expression for $\frac{d^2y}{dx^2}$ in terms of t ?

(A) $-\sin(t^2)$

(B) $-2t\sin(t^2)$

(C) $\cos(t^2) - 2t^2\sin(t^2)$

(D) $2\cos(t^2) - 4t^2\sin(t^2)$

19. Which of the following series is conditionally convergent?

(A) $\sum_{k=1}^{\infty}(-1)^k \frac{5}{k^3+1}$

(B) $\sum_{k=1}^{\infty}(-1)^k \frac{5}{k+1}$

(C) $\sum_{k=1}^{\infty}(-1)^k \frac{5k}{k+1}$

(D) $\sum_{k=1}^{\infty}(-1)^k \frac{5k^2}{k+1}$

20. Let f be the function defined by $f(x) = e^{2x}$. Which of the following is the Maclaurin series for f', the derivative of f ?

(A) $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!} + \cdots$

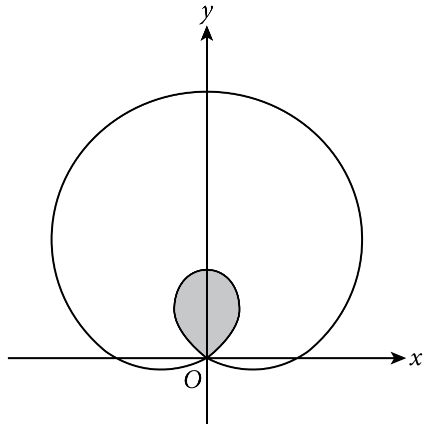
(B) $2 + 2x + \frac{2x^2}{2!} + \frac{2x^3}{3!} + \cdots + \frac{2x^n}{n!} + \cdots$

(C) $1 + 2x + \frac{(2x)^2}{2!} + \frac{(2x)^3}{3!} + \cdots + \frac{(2x)^n}{n!} + \cdots$

(D) $2 + 2(2x) + \frac{2(2x)^2}{2!} + \frac{2(2x)^3}{3!} + \cdots + \frac{2(2x)^n}{n!} + \cdots$

PART B (BC ONLY)

A graphing calculator is required on this part of the exam.



21. The figure above shows the graph of the polar curve $r = 2 + 4\sin\theta$. What is the area of the shaded region?

    (A) 2.174

    (B) 2.739

    (C) 13.660

    (D) 37.699

22. The function f has derivatives of all orders for all real numbers. It is known that $\left|f^{(4)}(x)\right| \le \frac{12}{5}$ and $\left|f^{(5)}(x)\right| \le \frac{3}{2}$ for $0 \le x \le 2$. Let $P_4(x)$ be the fourth-degree Taylor polynomial for f about $x = 0$. The Taylor series for f about $x = 0$ converges at $x = 2$. Of the following, which is the smallest value of k for which the Lagrange error bound guarantees that $\left|f(2) - P_4(2)\right| \le k$?

    (A) $\frac{2^5}{5!} \cdot \frac{3}{2}$

    (B) $\frac{2^5}{5!} \cdot \frac{12}{5}$

    (C) $\frac{2^4}{4!} \cdot \frac{3}{2}$

    (D) $\frac{2^4}{4!} \cdot \frac{12}{5}$