PART B (BC ONLY): Calculator not Permitted

4. The Maclaurin series for the function *f* is given by

$$f(x) = \sum_{k=1}^{\infty} \frac{(-1)^{k+1} x^k}{k^2} = x - \frac{x^2}{4} + \frac{x^3}{9} - \cdots$$ on its interval of convergence.

(a) Use the ratio test to determine the interval of convergence of the Maclaurin series for *f*. Show the work that leads to your answer.

(b) The Maclaurin series for *f* evaluated at $x = \frac{1}{4}$ is an alternating series whose terms decrease in absolute value to 0. The approximation for $f\left(\frac{1}{4}\right)$ using the first two nonzero terms of this series is $\frac{15}{64}$. Show that this approximation differs from $f\left(\frac{1}{4}\right)$ by less than $\frac{1}{500}$.

(c) Let *h* be the function defined by $h(x) = \int_0^x f(t)\,dt$. Write the first three nonzero terms and the general term of the Maclaurin series for *h*.

Part B: (BC ONLY): Calculator not Permitted
Scoring Guidelines for Question 4 — 9 points

Learning Objectives: LIM-7.A LIM-7.B LIM-8.D LIM-8.G

(a) Use the ratio test to determine the interval of convergence of the Maclaurin series for *f*. Show the work that leads to your answer.

Model Solution	Scoring	
$\lim_{k\to\infty}\left\|\dfrac{\frac{(-1)^{k+2}x^{k+1}}{(k+1)^2}}{\frac{(-1)^{k+1}x^k}{k^2}}\right\| = \lim_{k\to\infty}\dfrac{k^2}{(k+1)^2}\|x\| = \|x\|$	Sets up ratio	**1 point** 3.B
	Computes limit of ratio	**1 point** 1.E 4.C
$\|x\| < 1$ The series converges for $-1 < x < 1$.	Identifies interior or interval of convergence	**1 point** 3.D
When $x = -1$, the series is $\sum_{k=1}^{\infty}\frac{-1}{k^2}$. This is a convergent *p*-series.	Considers both endpoints	**1 point** 1.D
When $x = 1$, the series is $\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k^2}$. This series converges by the alternating series test. The interval of convergence of the Maclaurin series for *f* is $-1 \le x \le 1$.	Analysis and interval of convergence	**1 point** 3.D

Total for part (a) — **5 points**

(b) Show that this approximation differs from $f\left(\frac{1}{4}\right)$ by less than $\frac{1}{500}$.

$$\left|f\left(\frac{1}{4}\right) - \frac{15}{64}\right| < \frac{\left(\frac{1}{4}\right)^3}{9} = \frac{1}{576}$$

$$\frac{1}{576} < \frac{1}{500}$$

Scoring	
Uses third term as error bound	**1 point** 3.D
Error bound	**1 point** 3.E

Total for part (b) — **2 points**

(c) Write the first three nonzero terms and the general term of the Maclaurin series for *h*.

$$h(x) = \int_0^x f(t)\,dt = \frac{x^2}{2} - \frac{x^3}{12} + \frac{x^4}{36} - \cdots + \frac{(-1)^{k+1}x^{k+1}}{(k+1)k^2} + \cdots$$

First three nonzero terms; General term

Scoring	
First three nonzero terms	**1 point** 1.D
General term	**1 point** 1.D 4.C

Total for part (c) — **2 points**

Total for Question 4 — **9 points**