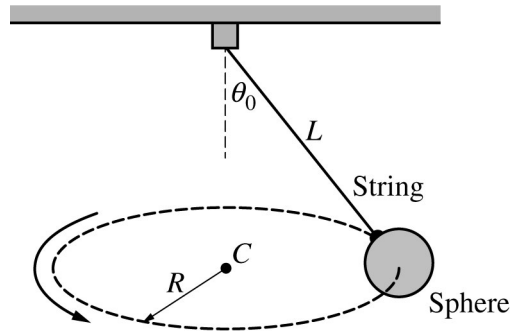## Question 1: Quantitative/Qualitative Translation

1. A small sphere of mass *M* is suspended by a string of length *L*. The sphere is made to move in a horizontal circle of radius *R* at a constant speed, as shown above. The center of the circle is labeled point *C*, and the string makes an θ_0 with the vertical.

   (A) Two students are discussing the motion of the sphere and make the following statements.

    Student 1: None of the forces exerted on the sphere are in the direction of point C, the center of the circular path. Therefore, I don't see how there can be a centripetal force exerted on the sphere to make it move in a circle.

    Student 2: I see another problem. The tension force exerted by the string is at an angle from the vertical. Therefore, its vertical component must be less than the weight Mg of the sphere. That means the net force on the sphere has a downward vertical component, and the sphere should move downward as well as moving around in a circle.

    i. What is one aspect of Student 1's reasoning that is incorrect?

    ii. What is one aspect of Student 2's reasoning that is incorrect?

    (B)

    i. Derive an equation for the magnitude of the net force exerted on the sphere. Express your answer in terms of M, theta, and physical constants, as appropriate.

    ii. Describe one aspect or step in your derivation of part (b)(i) that can be correctly linked to your answer to either part (a)(i) or part (a)(ii).
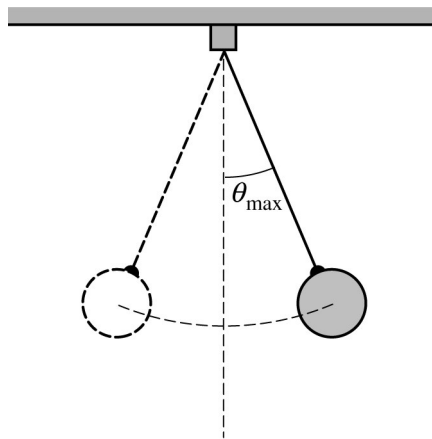
Instead of moving in a horizontal circle, the sphere now moves in a vertical plane so that it is a simple pendulum, as shown above. The maximum angle θ_{max} that the string makes from the vertical can be assumed to be small. The graph below shows data for the square of the pendulum period T as a function of string length L.
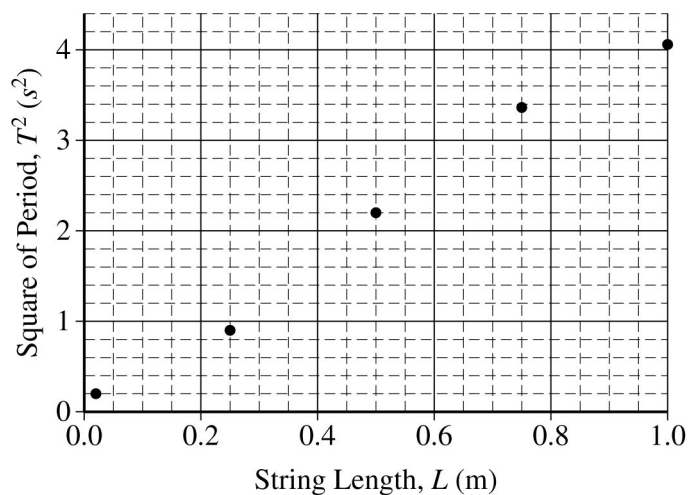
(C) On the graph above, draw a best-fit line for the data. Then use the line to calculate a numerical value for the gravitational acceleration g.
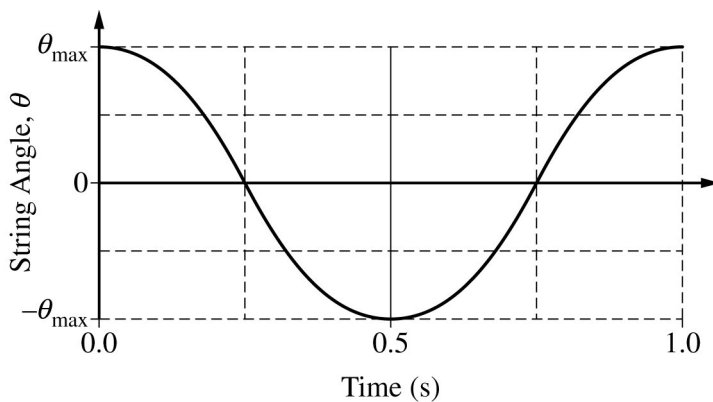
(D) The graph above shows the angle θ from the vertical as a function of time for the pendulum. On the axes below, sketch a graph of the gravitational potential energy of the sphere-Earth system for the same time interval. Take the zero of potential energy to be when the potential energy has its a minimum value.
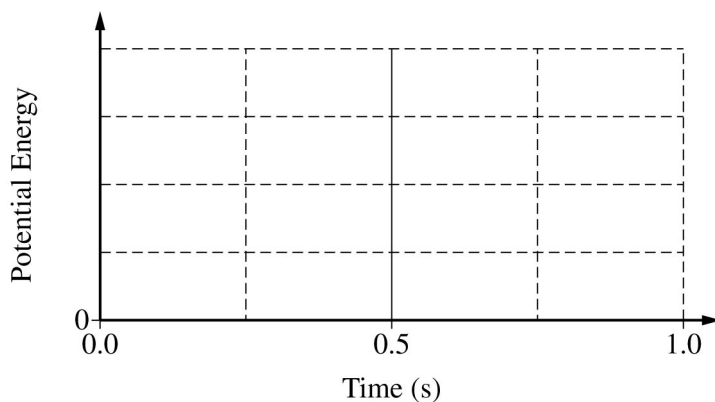
(E) As the sphere swings back and forth, it must also rotate a small amount during each swing. The figures below indicate the direction that the sphere rotates as it is swinging in each direction.
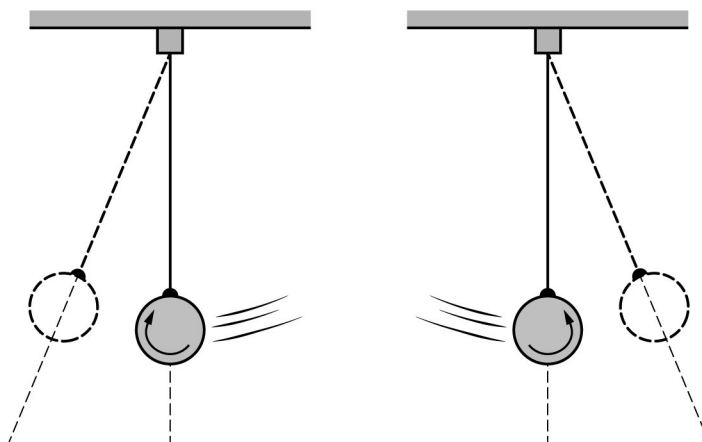
In order for the sphere's rotation to change direction, a torque must be exerted on the sphere. When the sphere is at its maximum <u>rightward</u> displacement, what is the direction of the torque exerted on the sphere with respect to the point of attachment between the sphere and string?

_____ Clockwise   _____ Counterclockwise

Briefly state why the torque is in the direction you indicated.

Scoring Guidelines for Question 1: Quantitative/Qualitative Translation

12 points

(A) **i.** What is one aspect of Student 1's reasoning that is incorrect?

1 point
6.4

One point for a claim (explicit or implied) that the sphere can have a centripetal force without any individual force pointing toward the center of its circular path.

Examples of an acceptable claim:

- The centripetal force is provided by the net force on an object.
- The tension force from the string has a horizontal component, which provides the centripetal force.

ii. What is one aspect of Student 2's reasoning that is incorrect?

1 point
6.4

One point for a claim (explicit or implied) that the tension force is or can be larger than Mg.

Examples of an acceptable claim:

- The tension force is larger than Mg.
- The tension force is larger than Mg, allowing the vertical component to be equal to Mg and the net force to be zero as it must be.

Total for part (A)	**2 points**

(B) **i.** Derive an equation for the magnitude of the net force exerted on the sphere, utilizing the terms appropriate terms.

1 point
1.4

One point for using Newton's second law for vertical force components.

$$\sum F_y = 0$$
$$F_T \cos\theta - Mg = 0 \text{ or } F_T \cos\theta = Mg$$

One point for writing the horizontal component of string tension in terms of angle.

1 point
1.4

$$F_{\text{net}} = F_x = F_T \sin\theta$$

One point for substituting a correct tension force and writing an answer in terms of the stated quantities.

1 point
2.2

$$F_T \cos\theta = Mg, \text{ so } F_T = Mg / \cos\theta$$
$$F_{\text{net}} = F_T \sin\theta = \left(\frac{Mg}{\cos\theta}\right)\sin\theta \text{ or } Mg\tan\theta$$

ii. Describe one aspect or step in your derivation of part (b)(i) that can be correctly linked to your answer to either part (a)(i) or part (a)(ii).

1 point
6.4

One point for a correct statement linking the derivation to the answer in either (A)(i) or (A)(ii).

Examples of an acceptable description:

- The net force F_{net} is in the horizontal direction and equals the horizontal component $F_T \sin\theta$, as indicated by the statement $F_{\text{net}} = F_T \sin\theta$ in the derivation.
- The tension force was shown to be $F_T = Mg / \cos\theta$, which is greater than Mg as stated in the answer to (B)(ii).

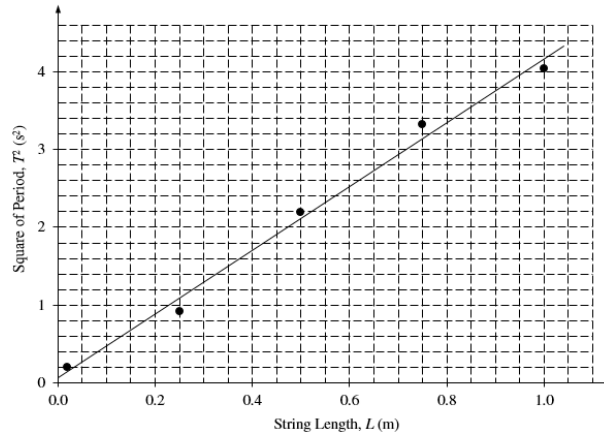|	
Total for part (B)	**4 points**

(C) Draw a best-fit line for the data.

1 point

One point for drawing a reasonable line of best fit. That is, the straight line drawn should have roughly the same amount of points above and below.

Example best fit line:

Then use the line to calculate a numerical value for the gravitational acceleration g.

1 point

2.2

One point for correctly calculating the slope or its inverse, using points on the line drawn.

$$\text{Slope} = \frac{\Delta(T^2)}{\Delta L} = \frac{4.15 - 1.3}{1.0 - 0.3} = 4.07 \ \text{s}^2/\text{m}$$

One point for a calculation of g consistent with the calculated slope or slope inverse.

1 point

2.2

$$T = 2\pi\sqrt{L/g} \quad \text{so} \quad T^2 = 4\pi^2 L/g$$
$$\text{Slope} = 4\pi^2/g$$
$$g = 4\pi^2/\text{Slope} = 4\pi^2/\left(4.07 \ \text{s}^2/\text{m}\right) = 9.7 \ \text{m}/\text{s}^2$$
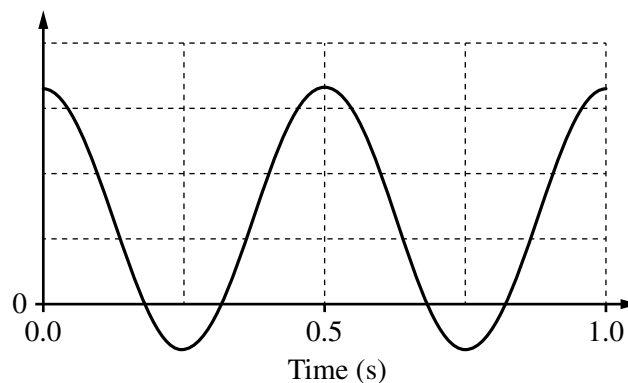
Total for part (C)   **3 points**

(D) Sketch a graph of the gravitational potential energy of the sphere-Earth system for the same time interval.

1 point

1.5

One point for a graph with equal maxima at 0, 0.5 s, and 1 s, and minima of zero at 0.25 s and 0.75 s.

One point for a graph that has even symmetry (mirror symmetry) about time.
$t = 0.5$ s

1 point

1.5

Notes:
- The maxima should be equal in energy, but may have any energy value.
- A graph with maxima at 0 and 1 s, and a minimum at 0.5 s, can earn the second point only.

Total for part (D)   **2 points**

(E) Briefly state why the torque is in the direction you indicated. **1 point**

One point for indicating that the torque is clockwise (claim) with acceptable reasoning that connects the claim with evidence. (Note: If the incorrect selection is made, the response is not graded.)

Examples of acceptable reasoning:

1. At the sphere's maximum rightward displacement, the sphere's rotation is changing from counterclockwise (swinging to the right) to clockwise (swinging to the left). So the torque must be clockwise.

2. At the sphere's maximum rightward displacement, the gravitational force (taken to act at the center of the sphere) exerts a clockwise torque about the point of attachment to the string.

Examples of acceptable evidence:

- The sphere is rotating counterclockwise when moving to the right, toward its maximum rightward displacement.
- The sphere is rotating clockwise when moving to the left, away from its maximum rightward displacement.
- The gravitational force is in the downward direction. Treated as acting at the center of the sphere, this force is exerted at a location (the sphere's center) that is to the right of the point of attachment.

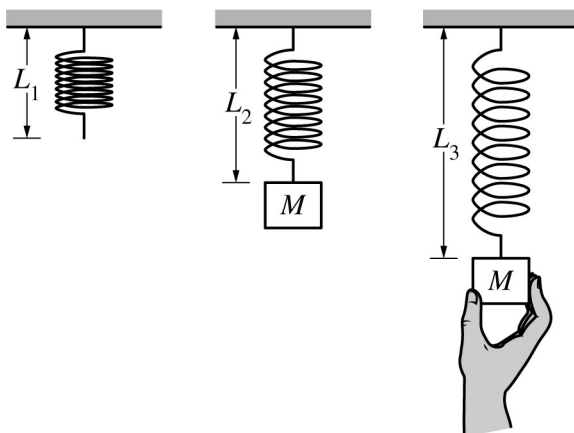|	
Total for part (E)	**1 point**
Total for question 1	**12 points**

Figure 1   Figure 2   Figure 3

2. A spring with unstretched length L_1 is hung vertically, with the top end fixed in place, as shown in Figure 1 above. A block of mass M is attached to the bottom of the spring, as shown in Figure 2, and the spring has length $L_2 > L_1$ when the block hangs at rest. The block is then pulled downward and held in place so that the spring is stretched to a length $L_3 > L_2$, as shown in Figure 3.

(A) On the dot below, which represents the block in Figure 3, draw and label the forces (not components) exerted on the block. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot.

(B) The student releases the block. Consider the time during which the block is moving upward toward its equilibrium position and the spring length is still longer than L_2.

In a clear, coherent paragraph-length response that may also contain diagrams and/or equations, indicate why the total mechanical energy is increasing, decreasing, or constant for each of the systems listed below.

- System 1: The block
- System 2: The block and the spring
- System 3: The block, the spring, and Earth

Use E_1, E_2, and E_3 to denote the total mechanical energy of systems 1, 2, and 3, respectively.

(A) Draw and label the forces (not components) exerted on the block.   **1 point**
                                                                                                                1.1

Accept the following:
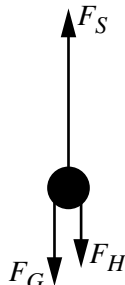
One point for including all three labelled forces exerted on the block:

- the upward spring force,
- the downward gravitational force,
- and the downward force of the hand holding the block.

One point for including at least one of the three forces exerted on the block, with no extraneous forces.   **1 point**
                                                                                                                1.1

Total for part (A)   **2 points**

(B) In a clear, coherent paragraph-length response that may also contain diagrams and/or equations,   **1 point**
indicate why the total mechanical energy is increasing, decreasing, or constant for each of the systems   6.4
listed below.

- System 1: The block
- System 2: The block and the spring
- System 3: The block, the spring, and Earth

One point for a response with no incorrect claims about which forms of energy are present in each system.

- Note: Responses that do not explicitly refer to the forms of energy in one or more of the systems can
  still earn this point.

The correct forms of energy in each system are the following:

- System 1: Kinetic energy K_{block} of the block (There is <u>no</u> potential energy for system 1.)
- System 2: K_{block} and the potential energy U_{spring} of the spring
- System 3: K_{block}, U_{spring}, and the gravitational potential energy U_{grav} of the block-Earth system

One point for correctly providing an indication that total mechanical energy is increasing for the block.   **1 point**
                                                                                                                1.4
Examples of acceptable statements:

- The block is accelerating upward with increasing speed.
- The net force on the block is upward, in the direction of the block's velocity, so the block's kinetic
  energy is increasing.

One point for correct statement of why the total mechanical energy is decreasing for the block-spring system.

1 point

2.1

Examples of acceptable reasoning statements:

- The only external forces exerted on the block-spring system (system 2) are the gravitational force on the block and the force holding the top end of the spring in place. The gravitational force is in the opposite direction of the block's motion and so does negative work on system 2. The point of application of the force on the top end of the spring does not move, so this force does zero work on the system. Overall, negative work is being done on system 2, so E_2 must be decreasing.

- System 2 has total mechanical energy:

  $E_2 = K_{block} + U_{spring}$, which equals $E_3 - U_{grav}$.

  E_3 is the total mechanical energy of System 3. E_3 is constant, while U_{grav} is increasing because the block is moving upward. So $E_2 = E_3 - U_{grav}$ must be decreasing.

One point for correct statement that the total mechanical energy is constant for a closed system such as the block-spring-Earth system.

1 point

6.4

Examples of acceptable claim statements:

- System 3 is closed, so its total mechanical energy remains constant.
- No external forces act on system 3, so its total mechanical energy remains constant.

Note: It is not necessary to state that energy is neither dissipated or added by converting other forms of energy into mechanical energy, since there is no mention of potential causes for these processes (e.g., friction to dissipate mechanical energy, or an explosion to add mechanical energy).

One point for a logical, relevant, and internally consistent argument that addresses the required argument, explanation or question asked.

1 point

1.4

Example of an acceptable response:

- For system 1 of just the block: When the force holding the block is removed, the block's acceleration and the net force on the block are upward since the spring force is greater than the gravitational force. The net force is in the direction of the block's motion, so its speed and kinetic energy are both increasing. The total mechanical energy is the kinetic energy for a single-object system, so E_1 is increasing.

  System 3 can be considered a closed system with no external forces exerted on it, so E_3 is constant. Note that:

  $$E_3 = K_{block} + U_{spring} + U_{grav'}$$

  where K_{block} is the block's kinetic energy, U_{spring} is the spring's potential energy, and U_{grav} is the gravitational potential energy of the block-Earth system.

  System 2 has total mechanical energy:

  $$E_2 = K_{block} + U_{spring}, \text{ which equals } E_3 - U_{grav}.$$

  E_3 is constant, while U_{grav} is increasing because the block is moving upward. So $E_2 = E_3 - U_{grav}$ must be decreasing.

Total for part (B)	**5 points**
Total for question 2	**7 points**