Scoring Guidelines for Free-Response Question 3 (12 points)

(a) (3 points)

(i) For indicating that the direction of the field vectors is the telling characteristic and describing how they indicate negative and positive charge — 1 point

Example: The direction of the field vectors. Field vectors near objects point toward negatively charged objects and away from positively charged objects.

(ii) For indicating that the size and distance of the closest field vectors are the telling characteristics — 1 point

For describing how the size and distance of the closest field vectors indicate the magnitudes of the charges — 1 point

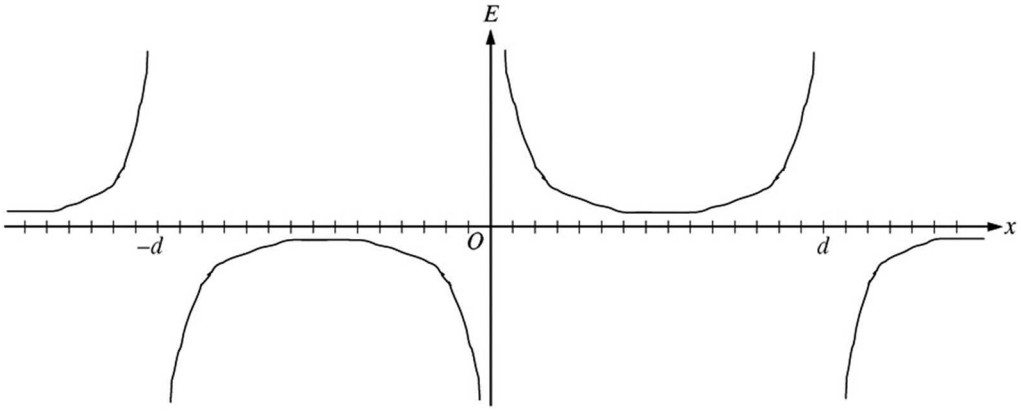
Field vectors nearest *R* and *T* are at about the same distance and have approximately the same length, so the magnitude of their charge is equal.

Field vectors nearest *S* have approximately the same length as field vectors nearest *R* and *T*, but the square of the distance to the field vectors nearest *S* is about twice the square of the distance to the field vectors nearest *R* and *T* –36 (6 tic marks squared) compared with 16 (4 tic marks squared). So the magnitude of the charge of *S* is twice the magnitude of the charges of *R* and *T*.

Example:

The vectors closest to ***R*** and ***T*** are about the same length and start at about the same distance. $q_R / d^2 = q_T / d^2$, so the charge on *R* is about the same as the charge on *T*. The closest vectors around *S* are about the same length as those around *R* and *T*. The vectors near *S* start at about 6 units away, while vectors near *R* and *T* start at about 4 units. $q_R / d^2 = q_s / \mathrm{D}^2$, so $q_s / \mathrm{q}_R = D^2 / \mathrm{d}^2 = 36/16 = 2.25$, , and so the charge on *S* is about twice that on *R* and *T*.

(b) (3 points)



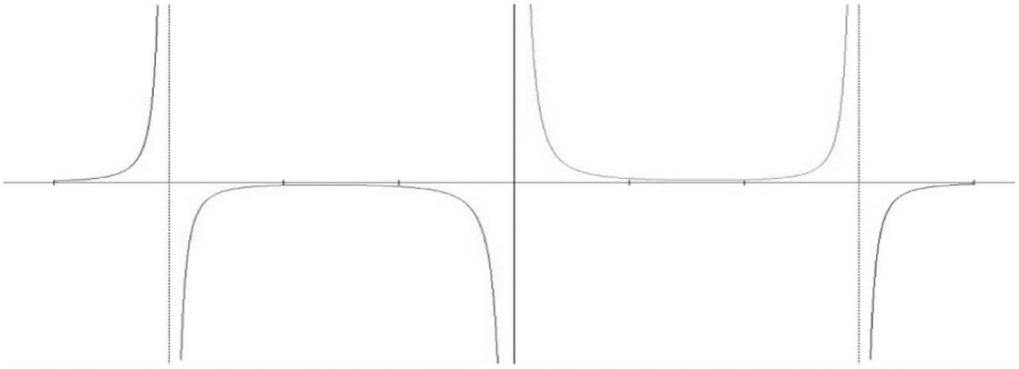
For showing reasonably close to asymptotic behavior on both sides of the charge positions — 1 point

For a general U shape between the charges that does not touch the *x*-axis — 1 point

Sample Questions

For all signs consistent with the vector map 1 point

The following shows a more exact plot. The scale is set to show the asymptotic behavior, which masks the nonzero values between charges and at both sides of the range.



(c) (3 points)

For correctly associating charge values with positions 1 point

For using the correct sign and power in the expressions for the distance to each charge 1 point

For the correct sign for the direction of the field for each term 1 point

$$E = k\left[-\frac{q}{(d-x)^2} \perp \frac{2q}{(x)^2} \perp \frac{q}{(d-x)^2}\right]$$

(d) (3 points)

For indicating that the value of the electric field is nonzero for all values between S and T 1 point

For justifying the above using one of the representations 1 point

For justifying the above using a second representation 1 point

The following are the ideas that should be expressed for each representation:

- The vectors between S and T in the electric field diagram all have nonzero length
- The graph between S and T never crosses the x-axis
- The negative term of the equation in part (c) is either always smaller than the other terms in this region or never completely cancels them both

Example:

The statement is not true. The vector diagram shows field vectors in this region with nonzero length, and the vectors not shown have even greater lengths.

The equation in part (c) shows that when $0 < x < d$, the denominator of the negative term is always greater than the denominator of the third term, but the numerator is the same. So the negative term always has a smaller magnitude than the third term, and since the second term is positive, the sum of the terms is always positive.

Note: A response claiming that there is a zero value can receive credit if it is consistent with an incorrect response in either part (b) or (c).