Scoring Guidelines

Part A (AB or BC): Graphing Calculator Required

t (hours)	0	2	4	6	8	10	12
$R(t)$ (vehicles per hour)	2935	3653	3442	3010	3604	1986	2201

1. On a certain weekday, the rate at which vehicles cross a bridge is modeled by the differentiable function R for $0 \le t \le 12$, where $R(t)$ is measured in vehicles per hour and t is the number of hours since 7:00 A.M. ($t = 0$). Values of $R(t)$ for selected values of t are given in the table above.

   (a) Use the data in the table to approximate $R'(5)$. Show the computations that lead to your answer. Using correct units, explain the meaning of $R'(5)$ in the context of the problem.

   (b) Use a midpoint sum with three subintervals of equal length indicated by the data in the table to approximate the value of $\int_0^{12} R(t)\,dt$. Indicate units of measure.

   (c) On a certain weekend day, the rate at which vehicles cross the bridge is modeled by the function H defined by $H(t) = -t^3 - 3t^2 + 288t + 1300$ for $0 \le t \le 17$, where $H(t)$ is measured in vehicles per hour and t is the number of hours since 7:00 A.M. ($t = 0$). According to this model, what is the average number of vehicles crossing the bridge per hour on the weekend day for $0 \le t \le 12$?

   (d) For $12 < t < 17$, $L(t)$, the local linear approximation to the function H given in part (c) at $t = 12$, is a better model for the rate at which vehicles cross the bridge on the weekend day. Use $L(t)$ to find the time t, for $12 < t < 17$, at which the rate of vehicles crossing the bridge is 2000 vehicles per hour. Show the work that leads to your answer.

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Part A (AB or BC): Graphing calculator required
Scoring Guidelines for Question 1 — 9 points

Learning Objectives: CHA-2.D CHA-3.A CHA-3.C CHA-3.F CHA-4.B LIM-5.A

(a) Use the data in the table to approximate $R'(5)$. Show the computations that lead to your answer. Using correct units, explain the meaning of $R'(5)$ in the context of the problem.

Model Solution	Scoring	
$R'(5) \approx \frac{R(6)-R(4)}{6-4} = \frac{3010-3442}{2} = -216$	Approximation using values from table.	**1 point** 2.B
At time $t = 5$ hours (12 P.M.), the rate at which vehicles cross the bridge is decreasing at a rate of approximately 216 vehicles per hour per hour.	Interpretation with units	**1 point** 3.F 4.B
	Total for part (a)	**2 points**

(b) Use a midpoint sum with three subintervals of equal length indicated by the data in the table to approximate the value of $\int_0^{12} R(t)\,dt$. Indicate units of measure

Model Solution	Scoring	
$\int_0^{12} R(t)\,dt \approx 4(R(2)+R(6)+R(10))$	Midpoint sum set up	**1 point** 1.E
$= 4(3653+3010+1986)$ $= 34{,}596$ vehicles	Approximation using values from the table with units	**1 point** 2.B 4.B
	Total for part (b)	**2 points**

(c) What is the average number of vehicles crossing the bridge per hour on the weekend day for $0 \le t \le 12$.

Model Solution	Scoring	
$\frac{1}{12-0}\boxed{\int_0^{12} H(t)\,dt} = 2452$ Definite integral — Answer	Definite integral	**1 point** 1.D 4.C
	Answer with supporting work	**1 point** 1.E
	Total for part (c)	**2 points**

(d) Use $L(t)$ to find the time t, for $12 \le t \le 17$, at which the rate of vehicles crossing the bridge is 2000 vehicles per hour. Show the work that leads to your answer.

Model Solution	Scoring	
$L(t) = H(12) - H'(12)(t-12)$ $H(12) = 2596,\ H'(12) = -216$	Slope	**1 point** 1.E 4.E
$L(t) = 2000$	$L(t) = 2000$	**1 point** 1.D
$\Rightarrow t = 14.759$	Answer with supporting work	**1 point** 1.E 4.E
	Total for part (d)	**3 points**

	Total for Question 1	**9 points**

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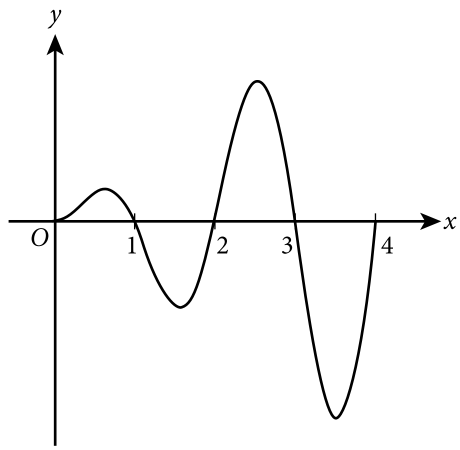
PART B (AB OR BC): Calculator not Permitted



Graph of f'

2. The figure above shows the graph of f', the derivative of a twice-differentiable function f, on the closed interval $[0, 4]$. The areas of the regions bounded by the graph of f' and the x-axis on the intervals $[0, 1]$, $[1, 2]$, $[2, 3]$, and $[3, 4]$ are 2, 6, 10, and 14, respectively. The graph of f' has horizontal tangents at $x = 0.6$, $x = 1.6$, $x = 2.5$, and $x = 3.5$. It is known that $f(2) = 5$.

   (a) On what open intervals contained in $(0, 4)$ is the graph of f both decreasing and concave down? Give a reason for your answer.

   (b) Find the absolute minimum value of f on the interval $[0, 4]$. Justify your answer.

   (c) Evaluate $\int_0^4 f(x) f'(x) dx$.

   (d) The function g is defined by $g(x) = x^3 f(x)$. Find $g'(2)$. Show the work that leads to your answer.

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Part A (AB or BC): Calculator not Permitted
Scoring Guidelines for Question 2 — 9 points

Learning Objectives: FUN-3.B FUN-4.A FUN-5.A FUN-6.D

(a) On what open intervals contained in (0,4) is the graph of f both decreasing and concave down? Give a reason for your answer.

Model Solution	Scoring	
The graph of f is decreasing and concave down on the intervals (1, 1.6) and (3, 3.5)	Answer	**1 point** 2.E
because f' is negative and decreasing on these intervals.	Reason	**1 point** 3.E 4.A
	Total for part (a)	**2 points**

(b) Find the absolute minimum value of f on the interval [0, 4]. Justify your answer.

Model Solution	Scoring	
The graph of f' changes from negative to positive only at $x = 2$.	Considers $x = 2$ as a candidate	**1 point** 3.B
$f(0) = f(2) + \int_2^0 f'(x)\,dx = f(2) - \int_0^2 f'(x)\,dx = 5 - (2-6) = 9$ $f(2) = 5$ $f(4) = f(2) + \int_2^4 f'(x)\,dx = 5 + (10 - 14) = 1$ On the interval [0, 4], the absolute minimum value of f is $f(4) = 1$.	Answer with justification	**1 point** 3.E
	Total for part (b)	**2 points**

(c) Evaluate $\int_0^4 f(x)f'(x)\,dx$

Model Solution	Scoring	
$\int_0^4 f(x)f'(x)\,dx = \frac{1}{2}(f(x))^2 \Big\vert_{x=0}^{x=4}$	Antiderivative of the form $a[f(x)]^2$	**1 point** 1.C
$= \frac{1}{2}\left((f(4))^2 - (f(0))^2\right)$	Earned the first point and $a = \frac{1}{2}$	**1 point** 1.E
$= \frac{1}{2}(1^2 - 9^2) = -40$	Answer	**1 point** 2.B
	Total for part (c)	**3 points**

(d) Find $g'(2)$. Show the work that leads to your answer.

Model Solution	Scoring	
$g'(x) = 3x^2 f(x) + x^3 f'(x)$ $g'(2) = 3 \cdot 2^2 f(2) + 2^3 f'(2) = 12 \cdot 5 + 8 \cdot 0 = 60$	Product Rule	**1 point** 1.E
	Answer	**1 point** 2.B
	Total for part (d)	**2 points**

	Total for Question 2	**9 points**

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PART A (BC ONLY): Graphing Calculator Required

3. For $0 \le t \le 5$, a particle is moving along a curve so that its position at time t is $(x(t), y(t))$. At time $t = 1$, the particle is at position $(2, -7)$. It is known that $\frac{dx}{dt} = \sin\left(\frac{t}{t+3}\right)$ and $\frac{dy}{dt} = e^{\cos t}$.

   (a) Write an equation for the line tangent to the curve at the point $(2, -7)$.

   (b) Find the y-coordinate of the position of the particle at time $t = 4$.

   (c) Find the total distance traveled by the particle from time $t = 1$ to time $t = 4$.

   (d) Find the time at which the speed of the particle is 2.5. Find the acceleration vector of the particle at this time.

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Part A (BC ONLY): Graphing Calculator Required
Scoring Guidelines for Question 3 — 9 points

Learning Objectives: CHA-3.G FUN-8.B

(a) Write an equation for the line tangent to the curve at the point (2, –7).

Model Solution	Scoring	
$\left.\dfrac{dy}{dx}\right\|_{t=1} = \left.\dfrac{\frac{dy}{dt}}{\frac{dx}{dt}}\right\|_{t=1} = \dfrac{e^{\cos 1}}{\sin\left(\frac{1}{4}\right)} = 6.938150$	Slope	**1 point** 1.C 4.E
An equation for the line tangent to the curve at the point (2, –7) is $y = -7 + 6.938(x-2)$.	Tangent line equation	**1 point** 1.D
	Total for part (a)	**2 points**

(b) Find the y-coordinate of the position of the particle at time $t = 4$.

Model Solution	Scoring	
$y(4) = -7 + \int_1^4 \frac{dy}{dt}\,dt = -5.006667$	Definite integral	**1 point** 1.D 4.C
The y-coordinate of the position of the particle at time $t = 4$ is –5.007 (or –5.006).	Answer	**1 point** 2.B
	Total for part (b)	**2 points**

(c) Find the total distance traveled by the particle from time $t = 1$ to time $t = 4$.

Model Solution	Scoring	
$\int_1^4 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt = 2.469242$	Definite integral	**1 point** 1.D 4.C
The total distance traveled by the particle from time $t = 1$ to time $t = 4$ is 2.469.	Answer	**1 point** 1.E 4.E
	Total for part (c)	**2 points**

(d) Find the time at which the speed of the particle is 2.5. Find the acceleration vector of the particle at this time.

Model Solution	Scoring	
$\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = 2.5 \Rightarrow t = 0.415007$	Speed equation	**1 point** 1.D 4.C
The speed of the particle is 2.5 at time $t = 0.415$.	Value of t	**1 point** 1.E 4.E
The acceleration vector of the particle at time $t = 0.415$ is: $\langle x''(0.415), y''(0.415)\rangle = \langle 0.255, -1.007\rangle$ (or $\langle 0.255, -1.006\rangle$).	Acceleration vector	**1 point** 1.E 4.E
	Total for part (d)	**3 points**

	Total for Question 3	**9 points**

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PART B (BC ONLY): Calculator not Permitted

4. The Maclaurin series for the function f is given by

$f(x) = \sum_{k=1}^{\infty} \frac{(-1)^{k+1} x^k}{k^2} = x - \frac{x^2}{4} + \frac{x^3}{9} - \cdots$ on its interval of convergence.

(a) Use the ratio test to determine the interval of convergence of the Maclaurin series for f. Show the work that leads to your answer.

(b) The Maclaurin series for f evaluated at $x = \frac{1}{4}$ is an alternating series whose terms decrease in absolute value to 0. The approximation for $f\left(\frac{1}{4}\right)$ using the first two nonzero terms of this series is $\frac{15}{64}$. Show that this approximation differs from $f\left(\frac{1}{4}\right)$ by less than $\frac{1}{500}$.

(c) Let h be the function defined by $h(x) = \int_0^x f(t)\,dt$. Write the first three nonzero terms and the general term of the Maclaurin series for h.

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Part B: (BC ONLY): Calculator not Permitted
Scoring Guidelines for Question 4 — 9 points

Learning Objectives: LIM-7.A LIM-7.B LIM-8.D LIM-8.G

(a) Use the ratio test to determine the interval of convergence of the Maclaurin series for *f*. Show the work that leads to your answer.

Model Solution	Scoring	
$$\lim_{k\to\infty}\left\lvert\frac{\frac{(-1)^{k+2}x^{k+1}}{(k+1)^2}}{\frac{(-1)^{k+1}x^k}{k^2}}\right\rvert = \lim_{k\to\infty}\frac{k^2}{(k+1)^2}\lvert x\rvert = \lvert x\rvert$$	Sets up ratio	**1 point** 3.B
	Computes limit of ratio	**1 point** 1.E 4.C
$\lvert x\rvert < 1$ The series converges for $-1 < x < 1$.	Identifies interior or interval of convergence	**1 point** 3.D
When $x = -1$, the series is $\sum_{k=1}^{\infty}\frac{-1}{k^2}$. This is a convergent *p*-series.	Considers both endpoints	**1 point** 1.D
When $x = 1$, the series is $\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k^2}$. This series converges by the alternating series test. The interval of convergence of the Maclaurin series for *f* is $-1 \le x \le 1$.	Analysis and interval of convergence	**1 point** 3.D

Total for part (a) — **5 points**

(b) Show that this approximation differs from $f\left(\frac{1}{4}\right)$ by less than $\frac{1}{500}$.

Model Solution	Scoring	
$$\left\lvert f\left(\frac{1}{4}\right) - \frac{15}{64}\right\rvert < \boxed{\frac{\left(\frac{1}{4}\right)^3}{9}} = \frac{1}{576}$$	Uses third term as error bound	**1 point** 3.D
$$\frac{1}{576} < \frac{1}{500}$$	Error bound	**1 point** 3.E

Total for part (b) — **2 points**

(c) Write the first three nonzero terms and the general term of the Maclaurin series for *h*.

$$h(x) = \int_0^x f(t)\,dt = \underbrace{\frac{x^2}{2} - \frac{x^3}{12} + \frac{x^4}{36}}_{\text{First three nonzero terms}} - \cdots + \underbrace{\frac{(-1)^{k+1}x^{k+1}}{(k+1)k^2}}_{\text{General term}} + \cdots$$

Scoring	
First three nonzero terms	**1 point** 1.D
General term	**1 point** 1.D 4.C

Total for part (c) — **2 points**

Total for Question 4 — **9 points**

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