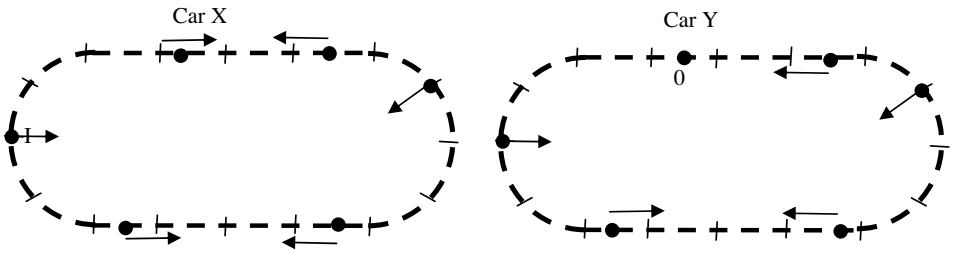
Scoring Guidelines for Free-Response Question 3 (12 points)

(a) (3 points)



For correct directions of the net forces at all the locations on the semicircular sections (i.e., all directed generally toward the center of the circle)	1 point
For correct directions of the net forces at all the locations on the bottom straightaways (i.e., directed toward the center of the segment)	1 point
For correct directions of the net forces at all locations on the top straightaway (i.e., both rightmost arrows directed toward the left, the left one for car *X* directed toward the right, and the left one for car *Y* equal to zero)	1 point

Return to Table of Contents

(b) (7 points total)

i) (2 points)

For realizing that the difference in time is only on the straightaways — 1 point

For correct reasoning leading to Car *Y* taking a shorter time on the straightaways — 1 point

Example:

Car *X* takes longer to accelerate and does not spend any time traveling at top speed. Car *Y* accelerates over a shorter time and spends time going at top speed. So car *Y* must cover the straightaways in a shorter time. Curves take the same time, so car *Y* must overall take a shorter time.

ii) (5 points)

The time to travel each curve is d/v_c. Answers can be expressed in terms of d/v_c or $t_c = d/v_c$ or some other defined unit of time. The calculations below will use $t_c = d/v_c$.

For stating that the time to travel each curve is d/v_c — 1 point

For correct kinematics expressions that allow determination of the time it takes for one segment of acceleration on the straightaways — 1 point

Example: $D = v_c t_1 + \frac{1}{2} a t_1^2, a = (2v_c - v_c)/t_1 = v_c/t_1$,

For work that shows an understanding of how to determine the time that car *X* and car *Y* each spend accelerating — 1 point

For work that shows an understanding of how to determine the time that car *Y* spends at constant speed — 1 point

For correctly determining the total straightaway times for each car — 1 point

Calculating the time for car *X* to travel one straightaway:

$$\frac{d}{2} = v_c t_1 + \frac{1}{2} a t_1^2, a = (2v_c - v_c)/t_1 = v_c/t_1$$

$$t_1 = \frac{d}{3v_c} = \frac{t_c}{3}, \text{total time is} \frac{2t_c}{3}$$

Calculating the time for car Y to travel one straightaway:

Doing the calculation shown above using the distance of acceleration $d/4$ gives the result that one section of acceleration takes a time $t_c/6$.

The time for car Y to travel one constant speed section on the straightaway is $(d/2)/2v_c = (t_c/4)$.

Adding three segments to get the total time for one straightaway gives $7t_c/12$.

The calculations show that car *Y* takes less time on a straightaway, and both cars take the same time on the curves, so car *Y* overall takes less time.

(c) (2 points)

For linking math to one aspect of qualitative reasoning that explains the difference in times — 1 point

For linking math to all other qualitative reasoning that explains the difference in times — 1 point

Examples:

The only difference in the calculations for the time of one segment of linear acceleration is the difference in distances. That shows that car *X* takes longer to accelerate. The equation $(d/2)/2v_c = (t_c/4)$ corresponds to car *Y* traveling for a time at top speed.

Substituting $a = v_c/t_1$ into the displacement equation in part (b) ii gives $D = (3/2)v_c t_1$. This shows that a car takes less time to reach its maximum speed when it accelerates over a shorter distance. This means car *Y* reaches its maximum speed more quickly and therefore spends more time at its maximum speed than car *X* does, as argued in part (b) i.